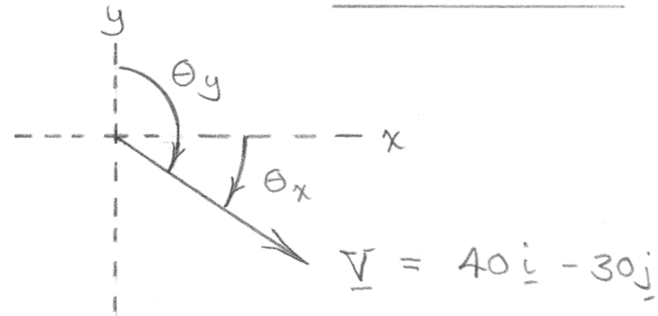
1/1 $V = \sqrt{V_x^2 + V_y^2} = \sqrt{40^2 + 30^2} = 50$

$$\underline{n} = \frac{\underline{V}}{V} = \frac{40\underline{i} - 30\underline{j}}{50} = \underline{0.8\underline{i} - 0.6\underline{j}}$$

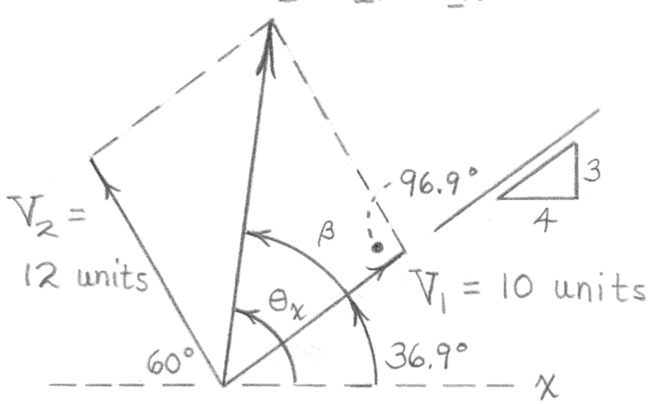
$\cos\theta_x = 0.8$, $\quad \underline{\theta_x = 36.9°}$

$\cos\theta_y = -0.6$, $\quad \underline{\theta_y = 126.9°}$



1/2

$\underline{V} = \underline{V}_1 + \underline{V}_2$ (But $V \neq V_1 + V_2$!!)



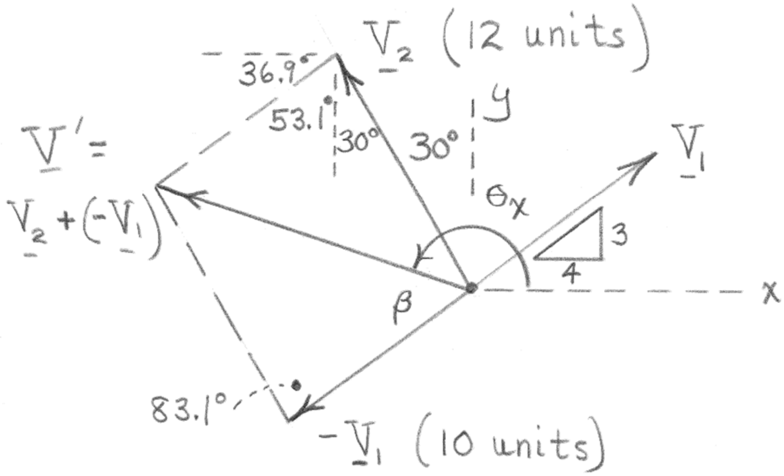
Graphically, $\underline{V = 16.4 \text{ units}, \quad \theta_x = 83°}$

Algebraically, $V^2 = 10^2 + 12^2 - 2(10)(12)\cos 96.9°$

$$\underline{V = 16.51 \text{ units}}$$

$\dfrac{\sin\beta}{12} = \dfrac{\sin 96.9°}{16.51}$, $\beta = 46.2°$

$\theta_x = \beta + 36.9° = 46.2° + 36.9° = \underline{83.0°}$



Graphically, $\underline{V' = 14.7 \text{ units}, \ \theta_x = 163°}$

Algebraically, $V'^2 = 10^2 + 12^2 - 2(10)(12)\cos 83.1°$

$$\underline{V' = 14.67 \text{ units}}$$

$\dfrac{\sin\beta}{12} = \dfrac{\sin 83.1°}{14.67}$, $\beta = 54.3°$

$\theta_x = (180° + 36.9°) - \beta = 180° + 36.9° - 54.3°$

$= \underline{162.6°}$

1/4

$$F = \sqrt{160^2 + 80^2 + 120^2} = 215 \text{ N}$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{160}{215} = 0.743, \qquad \underline{\theta_x = 42.0°}$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{80}{215} = 0.371, \qquad \underline{\theta_y = 68.2°}$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{-120}{215} = -0.557, \qquad \underline{\theta_z = 123.9°}$$

1/5

$$m = \frac{W}{g} = \frac{3000}{32.174} = \underline{93.2 \text{ slugs}}$$

$$m = 93.2 \text{ slugs} \left(\frac{14.594 \text{ kg}}{\text{slug}}\right) = \underline{1361 \text{ kg}}$$

from Table D/5

To illustrate the sensitivity of such calculations to significant-figure issues, we now use $g = 32.2$ ft/sec^2:

$$m = \frac{W}{g} = \frac{3000}{32.2} = 93.2 \text{ slugs} \checkmark$$

$$m = 93.2\,(14.594) = 1360 \text{ kg} \quad !$$

The value of $g = 32.2$ ft/sec^2 will normally, but not always, suffice.

1/6 $\quad F = W = \dfrac{G m_1 m_2}{r^2}$

where $\quad G = 6.673(10^{-11})\ m^3/(kg \cdot s^2)$

$m_1 = 85\ kg$

$m_2 = 5.976(10^{24})\ kg$

and $\quad r = (6371 + 250)(10^3)\ m$

Substitute these numbers & obtain $\underline{W = 773\ N}$

U.S. units : $W = 773\ N \left(\dfrac{1\ lb}{4.4482\ N}\right) = \underline{173.8\ lb}$

1/7

$$W = (125\ \text{lb})\left(\frac{4.4482\ \text{N}}{\text{lb}}\right) = \underline{556\ \text{N}}$$

$$m = \frac{W}{g} = \frac{125}{32.2} = \underline{3.88\ \text{slugs}}$$

$$m = \frac{W}{g} = \frac{556}{9.81} = \underline{56.7\ \text{kg}}$$

1/8

$A = 8.67, \quad B = 1.429$

$(A+B) = 8.67 + 1.429 = \underline{10.10}$

$(A-B) = 8.67 - 1.429 = \underline{7.24}$

$(AB) = (8.67)(1.429) = \underline{12.39}$

$(A/B) = 8.67/1.429 = \underline{6.07}$

1/9

$$F = \frac{G m_e m_m}{d^2} = \frac{6.673(10^{-11})(5.976 \cdot 10^{24})^2(1)(0.0123)}{(384\,398 \cdot 10^3)^2}$$

$$= \underline{1.984(10^{20})\ \text{N}}$$

$$F = 1.984(10^{20})\ \text{N}\left(\frac{1\ \text{lb}}{4.4482\ \text{N}}\right) = \underline{4.46(10^{19})\ \text{lb}}$$

1/10

$$\underline{F} = F\underline{n} = F\left(\frac{-4\underline{i}-2\underline{j}}{\sqrt{4^2+2^2}}\right),$$

where $F = \dfrac{G m_{cu} m_{st}}{d^2}$

$$= \frac{G\left(\rho_{cu}\frac{4}{3}\pi r^3\right)\left(\rho_{st}\frac{4}{3}\pi\left(\frac{r}{2}\right)^3\right)}{(4r)^2+(2r)^2}$$

$$= \frac{1}{90} G \rho_{cu} \rho_{st} \pi^2 r^4$$

$$= \frac{1}{90}\left(6.673\cdot 10^{-11}\right)(8910)(7830)\pi^2\, 0.050^4$$

$$= 3.19\left(10^{-9}\right) \text{ N}$$

Then $\underline{F} = 3.19\left(10^{-9}\right)\left[\dfrac{-4\underline{i}-2\underline{j}}{\sqrt{20}}\right]$

$$= \underline{\left(-2.85\underline{i} - 1.427\underline{j}\right)10^{-9} \text{ N}}$$

1/11 $E = 3 \sin^2\theta \tan\theta \cos\theta$

Exact: $E = 3 \sin^2 2° \tan 2° \cos 2°$

$= \underline{1.275(10^{-4})}$

Approx: $E_{ap} = 3(\theta^2)(\theta)(1)$

$= 3\theta^3$ (θ in rad)

$E_{ap} = 3\left[2\frac{\pi}{180}\right]^3 = \underline{1.276(10^{-4})}$

1/12 SI : $[Q] = (1)(kg)(m^2)/s^2$

$$= \underline{kg \cdot m^2/s^2}$$

U.S. : $[Q] = (1)(slug)(ft^2)/sec^2$

$$= \left(\frac{lb\text{-}sec^2}{ft}\right)(ft)^2/sec^2 = \underline{lb\text{-}ft}$$

Note: The SI units reduce to

$(kg \cdot m/s^2)\,m = N \cdot m$, but N is

not a base unit.

2/1

$$\begin{cases} F_x = 600 \cos 40° = \underline{460 \text{ N}} \\ F_y = -600 \sin 40° = \underline{-386 \text{ N}} \end{cases}$$

$$\underline{\mathbf{F} = 460\mathbf{i} - 386\mathbf{j} \text{ N}}$$

2/2

$$\underline{F} = 400\,(-\cos 30^\circ \underline{i} + \sin 30^\circ \underline{j})$$
$$= -346\underline{i} + 200\underline{j} \text{ N}$$

Scalar components: $\begin{cases} F_x = -346 \text{ N} \\ F_y = 200 \text{ N} \end{cases}$

Vector components: $\begin{cases} \underline{F}_x = -346\underline{i} \text{ N} \\ \underline{F}_y = 200\underline{j} \text{ N} \end{cases}$

2/3

$$\underline{F} = 6.5\left(-\frac{12}{13}\underline{i} - \frac{5}{13}\underline{j}\right)$$

$$= \overline{-6\underline{i} - 2.5\underline{j}\ \text{kN}}$$

(Note: Writing 6, rather than 6.00, indicates an exact result.)

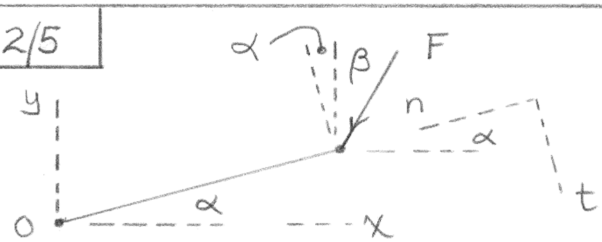
2/4

$$\underline{F} = F\underline{n}_{AB} = 34\left[\frac{15\underline{i} + 8\underline{j}}{\sqrt{15^2+8^2}}\right]$$

$$= 30\underline{i} + 16\underline{j} \text{ kN}$$

Scalar components: $\begin{cases} F_x = 30 \text{ kN} \\ F_y = 16 \text{ kN} \end{cases}$

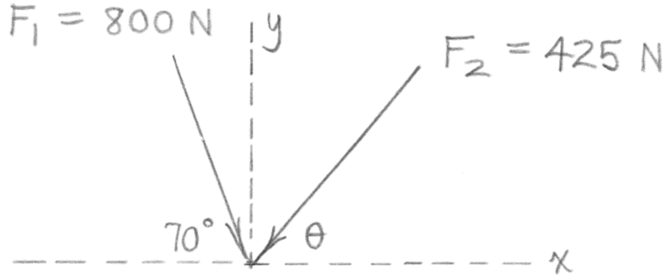
2/5



$$\begin{cases} F_x = -F\sin\beta \\ F_y = -F\cos\beta \end{cases}$$

$$\begin{cases} F_n = F\sin(\alpha+\beta) \\ F_t = F\cos(\alpha+\beta) \end{cases}$$

2/6



$$R_x = \Sigma F_x = 800 \cos 70° - 425 \cos\theta = 0$$

$$\underline{\theta = 49.9°}$$

$$R_y = \Sigma F_y = -800 \sin 70° - 425 \sin 49.9°$$

$$= -1077 \text{ N}$$

So $\underline{R = 1077 \text{ N}}$

2/7

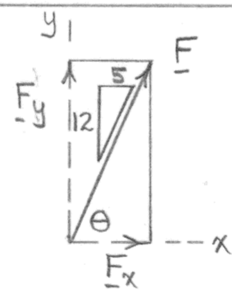
$$\mathbf{R} = (500 + 350\cos 60^\circ)\,\mathbf{i} + 350\sin 60^\circ\,\mathbf{j}$$

$$\underline{\mathbf{R} = 675\,\mathbf{i} + 303\,\mathbf{j}\ \text{N}}$$

$$R = \sqrt{675^2 + 303^2} \longrightarrow \underline{R = 740\ \text{N}}$$

$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right) = \cos^{-1}\left(\frac{675}{740}\right) \longrightarrow \underline{\theta_x = 24.2^\circ \text{ ABOVE } +x \text{ AXIS}}$$

2/8



$\cos\theta = \frac{5}{13}$, $\sin\theta = \frac{12}{13}$

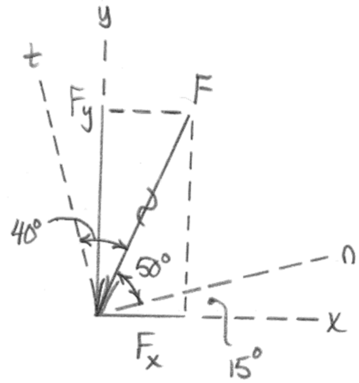
$F_y = F\sin\theta = F\frac{12}{13} = 320\text{ N}$

$\underline{F = 347\text{ N}}$

$F_x = F\cos\theta = 347\left(\frac{5}{13}\right) = \underline{133.3\text{ N}}$

2/9

$F = 65\ \text{kN}$

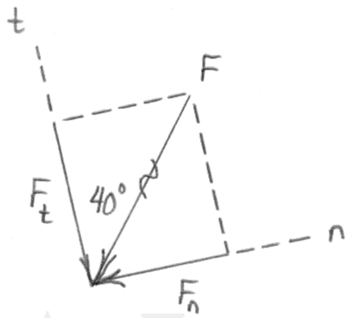


$F_x = -F\cos 65° = -65\cos 65°$

$\underline{F_x = -27.5\ \text{kN}}$

$F_y = -F\sin 65° = -65\sin 65°$

$\underline{F_y = -58.9\ \text{kN}}$



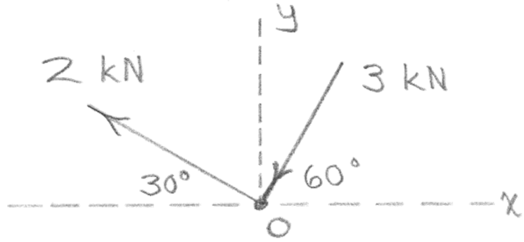
$F_n = -F\sin 40° = -65\sin 40°$

$\underline{F_n = -41.8\ \text{kN}}$

$F_t = -F\cos 40° = -65\cos 40°$

$\underline{F_t = -49.8\ \text{kN}}$

2/10



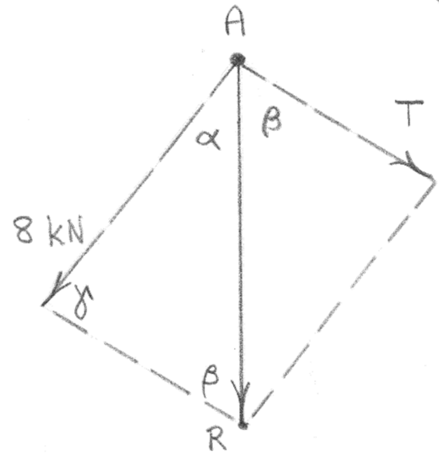
$$R_x = \Sigma F_x = -2\cos 30° - 3\cos 60° = -3.23 \text{ kN}$$

$$R_y = \Sigma F_y = 2\sin 30° - 3\sin 60° = -1.598 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \underline{3.61 \text{ kN}}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-1.598}{-3.23}\right) = \underline{206°}$$

2/11



$$\left\{\begin{array}{l}\alpha = \tan^{-1}\frac{40}{50} = 38.7^\circ \\ \beta = \tan^{-1}\frac{50}{30} = 59.0^\circ\end{array}\right.$$

$$\gamma = 180^\circ - \alpha - \beta$$

$$= 82.3^\circ$$

$$\frac{\sin\beta}{8} = \frac{\sin\alpha}{T}$$

$$\underline{T = 5.83 \text{ kN}}$$

$$\frac{\sin\beta}{8} = \frac{\sin\gamma}{R}, \quad \underline{R = 9.25 \text{ kN}}$$

2/12

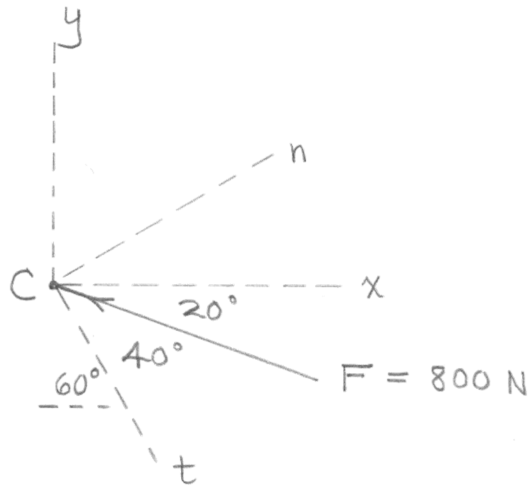
$$R_x = \Sigma F_x = 400 + 400 \cos 60° = 600 \text{ N}$$

$$R_y = \Sigma F_y = 400 \sin 60° = 346 \text{ N}$$

$$\Rightarrow \underline{R} = \underline{600\underline{i} + 346\underline{j} \text{ N}}$$

$$R = \sqrt{600^2 + 346^2} = \underline{693 \text{ N}}$$

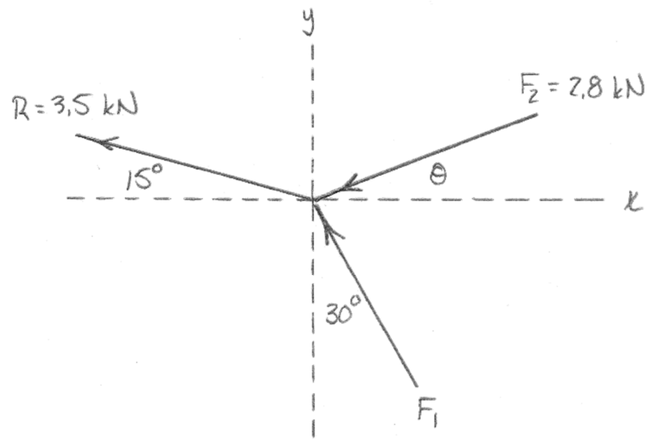
2/13



$$\begin{cases} F_x = -800\cos 20° = \underline{-752\text{ N}} \\ F_y = 800\sin 20° = \underline{274\text{ N}} \end{cases}$$

$$\begin{cases} F_n = -800\sin 40° = \underline{-514\text{ N}} \\ F_t = -800\cos 40° = \underline{-613\text{ N}} \end{cases}$$

2/14

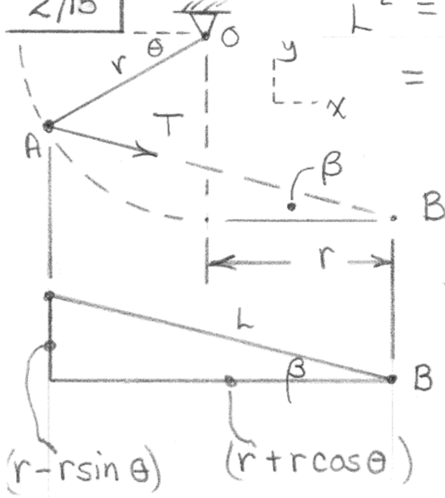


$$\begin{cases} R_x = \Sigma F_x : \quad -3.5 \cos 15° = -F_1 \sin 30° - 2.8 \cos \theta & (1) \\ R_y = \Sigma F_y : \quad 3.5 \sin 15° = F_1 \cos 30° - 2.8 \sin \theta & (2) \end{cases}$$

SOLVING (1) AND (2)...

$$\begin{cases} \underline{F_1 = 1.165 \text{ kN}} \\ \underline{\theta = 2.11°} \end{cases} \quad \text{OR} \quad \begin{cases} \underline{F_1 = 3.78 \text{ kN}} \\ \underline{\theta = 52.9°} \end{cases}$$

2/15



$$L^2 = (r - r\sin\theta)^2 + (r + r\cos\theta)^2$$

$$= r^2 - 2r^2\sin\theta + r^2\sin^2\theta$$

$$+ r^2 + 2r^2\cos\theta + r^2\cos^2\theta$$

$$= r^2(3 + 2\cos\theta - 2\sin\theta)$$

So $\cos\beta = \dfrac{r(1+\cos\theta)}{r\sqrt{3+2\cos\theta-2\sin\theta}}$

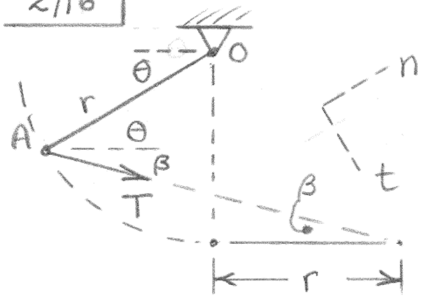
$$= \frac{1+\cos\theta}{\sqrt{3+2\cos\theta-2\sin\theta}}$$

$$\sin\beta = \frac{1-\sin\theta}{\sqrt{3+2\cos\theta-2\sin\theta}}$$

$$T_x = T\cos\beta = \underline{\frac{T(1+\cos\theta)}{\sqrt{3+2\cos\theta-2\sin\theta}}}$$

$$T_y = -T\sin\beta = \underline{\frac{T(\sin\theta-1)}{\sqrt{3+2\cos\theta-2\sin\theta}}}$$

2/16



From solution to previous problem:

$$\beta = \tan^{-1}\left[\frac{1-\sin\theta}{1+\cos\theta}\right]$$

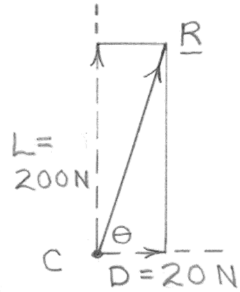
$$\begin{cases} T_n = T\cos(\theta+\beta) \\ T_t = T\sin(\theta+\beta) \end{cases}$$

For $T = 100$ N and $\theta = 35°$:

$\beta = 13.19°$

$$\begin{cases} T_n = 66.7 \text{ N} \\ T_t = 74.5 \text{ N} \end{cases}$$

2/17



$$\frac{L}{D} = \frac{200}{D} = 10, \quad D = 20 \text{ N}$$

$$R = \sqrt{L^2 + D^2} = \sqrt{(200)^2 + (20)^2}$$

$$= \underline{201 \text{ N}}$$

$$\theta = \tan^{-1}\left(\frac{L}{D}\right) = \tan^{-1}\left(\frac{200}{20}\right)$$

$$= \underline{84.3^\circ}$$

2/18 Using the coordinates of the problem figure:

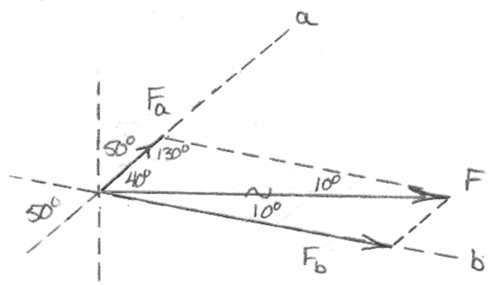
$$R_x = \Sigma F_x = 200 \cos 35° - 150 \sin 30°$$
$$= 88.8 \text{ N}$$

$$R_y = \Sigma F_y = 200 \sin 35° + 150 \cos 30°$$
$$= 245 \text{ N}$$

$$\therefore \underline{\mathbf{R} = 88.8\mathbf{i} + 245\mathbf{j} \text{ N}}$$

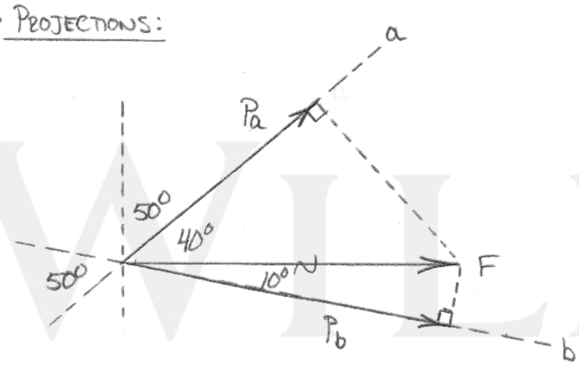
2/19	$F = 2.5$ kN

• COMPONENTS:



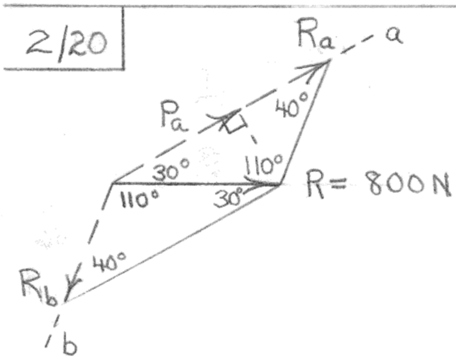
$$\frac{F}{\sin 130°} = \frac{F_a}{\sin 10°} = \frac{F_b}{\sin 40°} \longrightarrow \begin{cases} \underline{F_a = 0.567 \text{ kN}} \\ \underline{F_b = 2.10 \text{ kN}} \end{cases}$$

• PROJECTIONS:



$$\begin{cases} P_a = F\cos 40° = 2.5\cos 40° \longrightarrow \underline{P_a = 1.915 \text{ kN}} \\ P_b = F\cos 10° = 2.5\cos 10° \longrightarrow \underline{P_b = 2.46 \text{ kN}} \end{cases}$$

2/20



Law of sines :

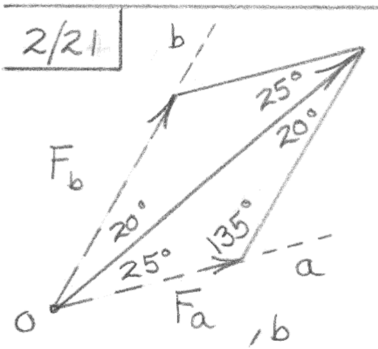
$$\frac{800}{\sin 40^\circ} = \frac{R_a}{\sin 110^\circ} = \frac{R_b}{\sin 30^\circ}$$

$\underline{R_a = 1170 \text{ N}}$

$\underline{R_b = 622 \text{ N}}$

Projection $P_a = R\cos 30^\circ = 800 \cos 30^\circ = \underline{693 \text{ N}}$

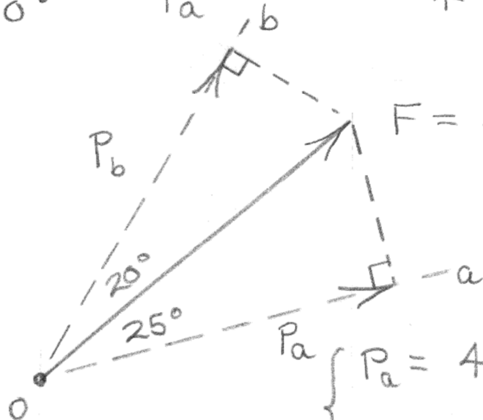
2/21



Components:

$$\frac{\sin 135^\circ}{4} = \frac{\sin 20^\circ}{F_a}, \quad \underline{F_a = 1.935 \text{ kN}}$$

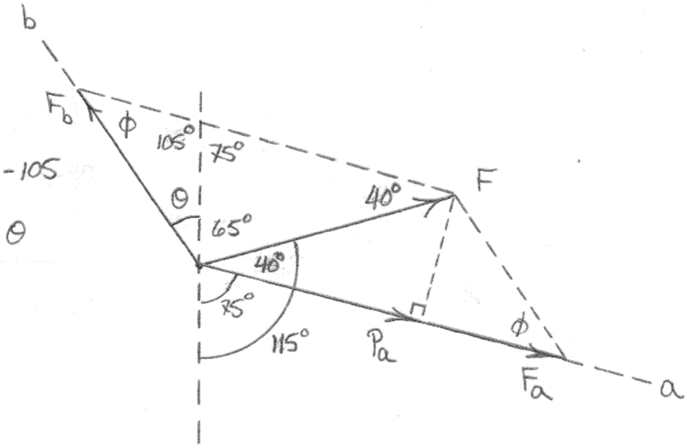
$$\frac{\sin 135^\circ}{4} = \frac{\sin 25^\circ}{F_b}, \quad \underline{F_b = 2.39 \text{ kN}}$$



Projections:

$$\begin{cases} P_a = 4 \cos 25^\circ = \underline{3.63 \text{ kN}} \\ P_b = 4 \cos 20^\circ = \underline{3.76 \text{ kN}} \end{cases}$$

2/22



$\phi = 180 - \theta - 105$

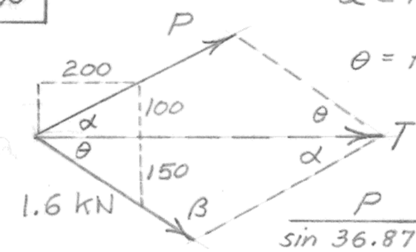
$\phi = 75° - \theta$

$P_a = F\cos 40° \longrightarrow 325 = F\cos 40° \longrightarrow \underline{F = 424 \text{ N}}$

$$\frac{F}{\sin\phi} = \frac{F_b}{\sin 40°} \longrightarrow \frac{424}{\sin(75°-\theta)} = \frac{325}{\sin 40°}$$

SOLVING THIS YIELDS... $\underline{\theta = 17.95° \text{ or } -48.0°}$

2/23



$$\alpha = \tan^{-1}\frac{100}{200} = 26.57^\circ$$

$$\theta = \tan^{-1}\frac{150}{200} = 36.87^\circ$$

$$\beta = 180 - (\alpha + \theta) = 116.57^\circ$$

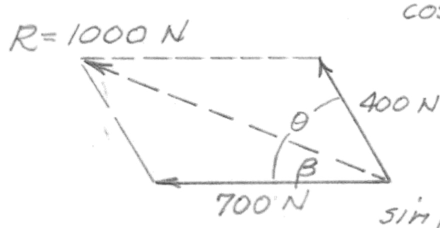
$$\frac{P}{\sin 36.87^\circ} = \frac{1.6}{\sin 26.57}$$

$$P = 1.6\,\frac{0.6}{0.4472} = \underline{2.15 \text{ kN}}$$

$$\frac{T}{\sin 116.57^\circ} = \frac{1.6}{\sin 26.57^\circ}, \quad T = 1.6\,\frac{0.8944}{0.4472} = \underline{3.20 \text{ kN}}$$

2/24 Law of cosines: $\overline{1000}^2 = \overline{400}^2 + \overline{700}^2 + 2(400)(700)\cos\theta$

$\cos\theta = 0.6250$, $\underline{\theta = 51.3^\circ}$



Law of sines:

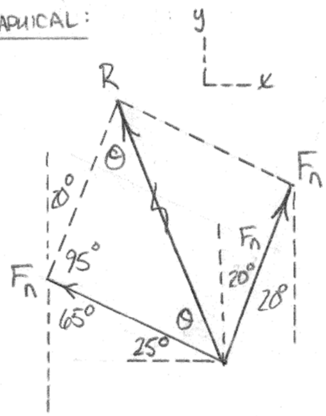
$$\frac{1000}{\sin(180^\circ - 51.3^\circ)} = \frac{400}{\sin\beta}$$

$$\sin\beta = \frac{400}{1000}\,0.7806 = 0.3122$$

$$\underline{\beta = 18.19^\circ}$$

2/25

• GRAPHICAL:



$$R = \sqrt{F_n^2 + F_n^2 - 2F_n^2 \cos 95^\circ}$$

$$R = \sqrt{2(5500)^2 [1 - \cos 95^\circ]}$$

$$R = 8110 \text{ N}$$

$$\theta = \frac{180^\circ - 95^\circ}{2} \longrightarrow \theta = 42.5^\circ$$

$R = 8110$ N @ 112.5° CCW FROM $+x$ AXIS

• VECTORS:

$$\underline{R} = (F_n \sin 20^\circ - F_n \sin 65^\circ)\,\underline{i} + (F_n \cos 20^\circ + F_n \cos 65^\circ)\,\underline{j}$$

$$\underline{R} = 5500\left[(\sin 20^\circ - \sin 65^\circ)\,\underline{i} + (\cos 20^\circ + \cos 65^\circ)\,\underline{j}\right]$$

$$\underline{R} = -3100\,\underline{i} + 7490\,\underline{j} \text{ N}$$

$$R = \sqrt{3100^2 + 7490^2} \longrightarrow R = 8110 \text{ N}$$

$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right) = \cos^{-1}\left(\frac{-3100}{8110}\right) \longrightarrow \theta_x = 112.5^\circ \text{ CCW FROM } +x \text{ AXIS}$$